Name Mohamed Raiyan Rizwan, ID 600093124

MTH 213, Quiz 1

15/15

Ayman Badawi

QUESTION 1. Let x be the number of balls in a basket. Given $1 \le x < 88$. Given $x \ (mod\ 8) = 5$ and $x \ (mod\ 11) = 4$. Use the CRT and find the value of x.

$x \ (\text{mod } 8) = 5$

$x \ (\text{mod } 11) = 4$

Is $\gcd(8, 11) = 1$? Yes x is unique

$m_1 = 8$ $m_2 = 11$ $m = 88$

$n_1 = \frac{m}{m_1} = 11$

$n_2 = \frac{m}{m_2} = 8$

multiplicative inverse

$11 y_1 = 1 \ (\text{mod } 8)$

$3 y_1 = 1$ $\quad y_1 = 3$

$8 y_2 = 1 \ (\text{mod } 11)$

$y_2 = 7$

$x = (r_1 n_1 y_1 + r_2 n_2 y_2) \ (\text{mod } m)$

$x = (5 \times 11 \times 3 + 4 \times 8 \times 7) \ (\text{mod } \cancel{m}\ 88)$

$x = 37$

6/6

QUESTION 2. Solve for x, $6x = 3$ over planet Z_9.

$6x = 3$ $\quad Z_9$ $\quad a = 6$ $\quad b = 3$ $\quad n = 9$

Is $\gcd(a, n) \mid b$? $\to$ $3 \mid 3$? Yes $\therefore$ 3 solutions

$\cancel{x = \{}$ $\quad x = \{5, 8, 2\}$ 5/5

QUESTION 3. Let $n = (10)^2(2)^3$.

(i) Find $\phi(n)$.

$n = 5^2 \times 2^2 \times 2^3 = \cancel{5^2 \times 4}\ 5^2 \times 2^5$

$\phi(n) = \left[q_1^{\alpha_1 - 1} (q_1 - 1) \right] \times \left[q_2^{\alpha_2 - 1} (q_2 - 1) \right]$

$\phi(n) = 5^1 (4) \times 2^4 (1)$

$= 320$

2/2

(ii) Find $11^{643} \ (mod\ n)$

$a^{\phi(n)} (\text{mod } n) = 1$

$\cancel{11^{320} \text{ mo}}$ $\quad 11^{643} (\text{mod } n)$

$= 11^{320} \times 11^{320} \times 11^{3} \ (\text{mod } n)$

$11^{320} (\text{mod } n) = 1$

$\therefore \to = 11^3 \ (\text{mod } 10^2 2^3)$

$11^{643} (\text{mod } n) = 531$ 2/2

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Name Mohammad Arfin Ameen, ID 93780

15/15

MTH 213, Quiz 2

Ayman Badawi

QUESTION 1. Prove directly that 4 is a factor of $n^2 + 2n + 9$ for every **odd** integer $n \in Z$;i.e., show that $n^2 + 2n + 9 = 4k$ for some integer $k \in Z$.

Proof (Direct): Let $n = 2m+1$ (odd no.), $m \in Z$

$\therefore n^2 + 2n + 9 = (2m+1)^2 + 2(2m+1) + 9 = 4m^2 + 1 + 4m + 4m + 2 + 9$

$= 4m^2 + 8m + 12$

$= 4(m^2 + 2m + 3)$

$= 4K_1$

5/5

Let $K_1 = (m^2 + 2m + 3)$ where $K_1 \in Z$

$\therefore n^2 + 2n + 9 = 4K_1$

Hence Proved.

QUESTION 2. Prove directly that $n^2 + 5n$ is an even integer for every $n \in Z$.

Proof (direct): Case I: Let n be even

Let $n = 2m$ (even no.), $m \in Z$

5/5

$n^2 + 5n = (2m)^2 + 5(2m) = 4m^2 + 10m = 2(2m^2 + 5m)$

$= 2K_1$, $K_1 = 2m^2 + 5m$, $K_1 \in Z$.

$\therefore n^2 + 5n$ is even when n is even

Case II: Let n be odd

$\therefore$ Let $n = 2m_2 + 1$ (odd no.) $n^2 + 5n = (2m_2+1)^2 + 5(2m_2+1) = 4m_2^2 + 4m_2 + 1 + 10m_2 + 5$

$= 4m_2^2 + 14m_2 + 6$

$= 2(2m_2^2 + 7m_2 + 3)$

Let $K_2 = 2m_2^2 + 7m_2 + 3$, $K_2 \in Z$

$= 2K_2$

$\therefore n^2 + 5n$ is even when n is odd.

$\therefore n^2 + 5n$ is even for all $n \in Z$

Hence Proved

QUESTION 3. Use the 4^{th}-method (contradiction) and prove that $\sqrt{13}$ is irrational.

Proof: (Contradiction) Assume $\sqrt{13}$ is rational.

$\therefore (\sqrt{13})^2 = \left(\frac{a}{b}\right)^2$, where $a \in Z, b \in Z^*$ and $\gcd(a,b) = 1$

$\Rightarrow 13 = \frac{a^2}{b^2} \Rightarrow 13b^2 = a^2$

$\because$ 13 is odd, we assume a is odd and b is odd

$\therefore$ Let $a = 2K_1 + 1$, $K_1 \in Z$

5/5

$b = 2K_2 + 1$, $K_2 \in Z$

$13(2K_2+1)^2 = (2K_1+1)^2 \Rightarrow 4 \cdot 13K_2^2 + 4 \cdot 13K_2 + 13 = 4K_1^2 + 4K_1 + 1$

(using 4th method) $\frac{4 \cdot 13K_2^2 + 4 \cdot 13K_2 + 12}{4} = \frac{4K_1^2 + 4K_1}{4}$

We know that:

$\boxed{\begin{array}{l} \bullet \text{ odd} + \text{even} = \text{odd} \\ \bullet\ cn^2 + cn = \text{even for } n \in Z \\ \bullet\ n^2 + n = \text{even for } n \in Z \end{array}}$

$\Rightarrow \underbrace{\underbrace{13K_2^2 + 13K_2}_{\text{even}} + \underbrace{3}_{\text{odd}}}_{\text{odd}} = \underbrace{K_1^2 + K_1}_{\text{even}}$

odd = even, which is not possible, contradiction

$\therefore$ Our assumption is wrong $\therefore \sqrt{13}$ is irrational by contradiction

Name Sneha Athota , ID 93063

15/15

MTH 213, Quiz 3

Ayman Badawi

8/8

QUESTION 1. Use Math. Induction and prove that $9 \mid (4^{3n} - 1)$, for every integer $n \geq 1$, where $n \in \mathbb{Z}^+$.

① Let's prove $9 \mid (4^{3n}-1)$ for $n=1$,

1 $4^{3(1)}-1 = 4^3-1 = 63 = 9(7)$ ✓

1 ② Let's assume, $4^{3n}-1 = 9k$ for some $n, n \geq 1$ & $k \in \mathbb{Z}$

③ To prove: $4^{3(n+1)}-1 = 9m$, where $m \in \mathbb{Z}$

6 Proof: $4^{3n+3}-1 = 4^{3n} \cdot 4^3 + 4^3 - 4^3 - 1$

$= 4^3[4^{3n}-1] + 4^3 - 1$

$= 4^3(9k) + 9(7)$ [from ① & ②]

$= 9[4^3 k + 7]$

$= 9m$ where $m = 4^3k + 7 \in \mathbb{Z}$

7/7

QUESTION 2. Use Math. Induction and prove that $1 + 2 + 2^2 + \cdots + 2^n = 2^{(n+1)} - 1$, for every integer $n \geq 1$, where $n \in \mathbb{Z}^+$.

① Prove $1+2+2^2+\cdots+2^n = 2^{(n+1)}-1$ for $n=1$,

$2^0+2^1 = 1+2 = 3$

1 $2^{(n+1)}-1 = 2^2-1 = 3$

$\therefore 1+2+\cdots+2^n = 2^{(n+1)}-1$ is true for $n=1$.

1 ② Assume, $1+2+2^2+\cdots+2^n = 2^{(n+1)}-1$ for some $n \in \mathbb{Z}^+$ & $n \geq 1$.

5 ③ To prove: $1+2+\cdots+2^n+2^{(n+1)} = 2^{(n+2)}-1$

Proof: $1+2+2^2+\cdots+2^n+2^{(n+1)} = 2^{(n+1)}-1+2^{(n+1)}$ [from ②]

$= 2 \cdot 2^{(n+1)} - 1$

$= 2^{n+1+1} - 1$

$= 2^{n+2} - 1$

Hence, proved

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Name Joud Hamdan , ID 95608

MTH 213, Quiz 4

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15/15

QUESTION 1. Use TRUTH table and prove that

$$(S_1 + S_2) \to S_3 \equiv (S_1 \to S_3) \cdot (S_2 \to S_3)$$

X

same.

S_1	S_2	S_3	S_1+S_2	$S_1 \to S_3$	$S_2 \to S_3$	$(S_1+S_2)\to S_3$	$(S_1\to S_3)\cdot(S_2\to S_3)$
1	1	1	1	1	1	1 ✓	1 ✓
1	1	0	1	0	0	0 ✓	0 ✓
1	0	1	1	1	1	1 ✓	1 ✓
1	0	0	1	0	1	0 ✓	0 ✓
0	1	1	1	1	1	1 ✓	1 ✓
0	1	0	1	1	0	0 ✓	0 ✓
0	0	1	0	1	1	1 ✓	1 ✓
0	0	0	0	1	1	1 ✓	1 ✓

5/5

QUESTION 2. Write down T or F $y^2 = x^2$

(i) $\exists x \in N^*$ such that $\forall y \in Z$, we have $y^2 - x^2 = 0$ **F**

$(x-4)(x+1)=0$
$x=4$ or $x=-1$
$\in N$ $\notin N$

(ii) $\exists! x \in N$ such that $x^2 - 3x - 4 = 0$. **T**

4/4

(iii) If $\underbrace{x^2 + 4 = 0}_{S_1}$ for some $x \in Z$, then $\underbrace{x^2 + 6 = 2}_{S_2}$ **T**

$S_1 = F$
$S_2 = F$
$S_1 \Rightarrow S_2$ ✓

(iv) $\forall y \in Z^*, \exists! x \in Q$ such that $xy = 2023$ **T**

QUESTION 3.

$For\ k = 4\ to\ \ n^3 + 3\ \ do$
$\quad x = k^3 + 2 * k^2 + 7$
$\quad For\ i = 1\ to\ \ 6k\ \ do$
$\quad\quad y = i^2 + 7 * i + k^2$
$\quad Next\ i$
$Next\ k$

a) Find the exact number of arithmetic operations that are executed by the code.

Outer loop	inner loop
will run $n^3+3-4+1$	will run $6k-1+1 = 6k$ times $\forall k$
$= n^3$ times	$k=4$: $\to 6(4)(5) = 120$ operations
$6n^3$ arithmetic operation executed ✓	$k = n^3+3$: $6(n^3+3) \times 5 = 5(6n^3+18)$ operations ✓

4/4

\# of arithmetic operations executed by code $= 6n^3 + \left[\dfrac{120 + 5(6n^3+18)}{2}\right](n^3)$ ✓

(b) Find $O(code)$ (i.e., complexity of the code)

$O(n^6)$

2/2

Name Gayatri Lakshmi, ID 92193

15/15

MTH 213, Quiz 5

Ayman Badawi

QUESTION 1. Let $A = \{1, \{2\}, 2, 5, 8\}$ and $B = \{3, 1, 8\}$. Then

(i) $A - B =$ $\{\{2\}, 2, 5\}$

(ii) $|AXB| =$ $5 \times 3 = 15$

(iii) $|P(A)| =$ $2^5 = 32$

(iv) Write down T or F

a. $\{2, 5\} \subset P(A)$. False

b. $\{\{2\}, 8\} \in P(A)$ True

c. $(\{3\}, \{8\}) \in BXA$ False

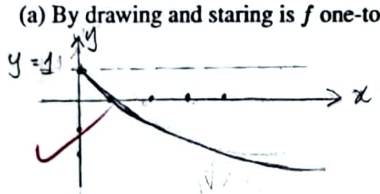
d. $\{2, \{2\}, 5\} \in P(A)$ True

e. $(3, 2) \in AXB$ False

f. $\{(3, \{2\})\} \in P(BXA)$ True

QUESTION 2. Let $f : [0, \infty) \rightarrow]-\infty, \cancel{0}1[$ such that $f(x) = -\sqrt{x} + 1$

(a) By drawing and staring is f one-to-one and ONTO?



By staring f is one to one and onto.
$\therefore$ inverse exists

(b) If f has an inverse, find the domain and the co-domain of f^{-1}, then find the equation of f^{-1}.

$f^{-1} :]-\infty, 1] \rightarrow [0, \infty)$

$y = -\sqrt{x} + 1$

$x = -\sqrt{y} + 1$

$x - 1 = -\sqrt{y} \Rightarrow \sqrt{y} = 1 - x$

$y = (1-x)^2 \quad \therefore f^{-1}(x) = (1-x)^2$

QUESTION 3. Consider the following permutation function

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 8 & 3 & 4 & 2 & 6 & 7 & 9 & 1 & 10 & 5 \end{pmatrix}$$

Find the smallest n such that $f^n = I$.

disjoint sets = $(1\,8)(2\,3\,4)(5\,6\,7\,9\,10)$
2 cycles 3 cycles 5 cycles

=

n = LCM $(2, 3, 5)$ = 30 cycles

$2 \,|\, 2, 3, 5$
$ \,|\, 1, 3, 5$
$= 1 \times 2 \times 3 \times 5 = 30$

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MTH 213, Quiz 6

Ayman Badawi

15/15

QUESTION 1. Given $a_n = a_{n-1} + 6a_{n-2}$, where $a_1 = 4, a_2 = 22$. Find the general formula for a_n [you may use a calculator to find c_1, c_2]

1) Rearrange: $a_n - a_{n-1} - 6a_{n-2} = 0$

2) Find Homogenous a_h: $a_n = \alpha^n \Rightarrow$

$\alpha^n - \alpha^{n-1} - 6\alpha^{n-2} = 0$ Divide by α^{n-2}

$\alpha^2 - \alpha - 6 = 0$ Solving

$\alpha = 3, \alpha = -2$

$\therefore a_h = c_1(3)^n + c_2(-2)^n$

3) $a_p = 0$

4) $a_n = c_1(3)^n + C_2(-2)^n$

Solving c_1 and C_2:

$n = 1$: $c_1(3) + C_2(-2) = 4 \Rightarrow 3c_1 - 2C_2 = 4$

$n = 2$: $c_1(3)^2 + C_2(-2)^2 = 22 \Rightarrow 9c_1 + 4C_2 = 22$

Solving: $c_1 = 2$ $C_2 = 1$

4/4

$\therefore$ $\boxed{\text{General Formula: } a_n = 2(3)^n + 1(-2)^n}$

To check: By recurrance: $a_3 = 22 + 6 \times 4 = 46$

By Formula: $a_3 = 2(3)^3 - 8 = 46$

QUESTION 2. Given $a_n = 4a_{n-1} + 5a_{n-2} + \underbrace{6n + 5}_{\text{partular}}$. Find the general formula for a_n [you don't need to find c_1, c_2]

1) Rearrange: $a_n - 4a_{n-1} - 5a_{n-2} = 6n + 5$

2) Homogeneous $a_h \Rightarrow a_n - 4a_{n-1} - 5a_{n-2} = 0$

$a_n = \alpha^n \Rightarrow \alpha^n - 4\alpha^{n-1} - 5\alpha^{n-2} = 0$

Divide by α^{n-2}

$\Rightarrow \alpha^2 - 4\alpha - 5 = 0$ Solving: $\alpha = 5, \alpha = -1$

$a_h = c_1(5)^n + C_2(-1)^n$ 3/3

3) a_p: Particular:

$a_p(n) - 4a_p(n-1) - 5a_p(n-2) = 6n + 5$

$(bn + c) - 4(b(n-1) + c) - 5(b(n-2) + c) = 6n + 5$

$\cancel{bn + c - 4bn + 4b - 4c - 5bn} + 10b - 5c = 6n + 5$

$(b - 4b - 5b)n + (c + 4b - 4c + 10b - 5c) = 6n + 5$

$\therefore -8b = 6$ $b = -\frac{6}{8} = \frac{-3}{4}$

6/6

$-8c + 14b = 5$

$-8c = 5 + 14\left(\frac{+3}{4}\right)$

$-8c = 5 + \frac{21}{2}$

$c = -\frac{31}{16}$ $\therefore$ $a_p = \frac{-3}{4}n - \frac{31}{16}$

4.) $a_n = a_h + a_p$

$\boxed{a_n = c_1(5)^n + C_2(-1)^n - \frac{3}{4}n - \frac{31}{16}}$ 3/3

QUESTION 3. Given $a_n = 4a_{n-1} + 5a_{n-2} + 12(3^n)$. Find the general formula for a_n [you may use the $a_h(n)$ from question 2, also you don't need to find c_1, c_2]

1) Rearrange: $a_n - 4a_{n-1} - 5a_{n-2} = 12(3^n)$

2) Homogenous: a_h: $a_n - 4a_{n-1} - 5a_{n-2} = 0$

$a_n = \alpha^n$: $\alpha^n - 4\alpha^{n-1} - 5\alpha^{n-2} = 0$

$\Rightarrow$ Divide by α^{n-2}:

$\alpha^2 - 4\alpha - 5 = 0$ $\alpha = 5, \alpha = -1$

$a_h = c_1(5)^n + c_2(-1)^n$

3) A_p: Particular

$a_p(n) = A(3)^n$

Substituting

$a_p(n) - 4a_p(n-1) - 5a_p(n-2) = 12(3)^n$

$A(3)^n - 4A(3)^{n-1} - 5A(3)^{n-2} = 12(3)^n$

Divide by 3^n

$A - \frac{4}{3}A - \frac{5}{9}A = 12$

$-\frac{8}{9}A = 12$

$A = -\frac{12 \times 9}{8} = -\frac{27}{2}$ $a_p(n) = \frac{-27}{2}(3$

4) $a_n = a_h + a_p$

$\boxed{a_n = c_1(5)^n + C_2(-1)^n - \frac{27}{2}(3)^n}$ 4/4

Name Afra AlFalahi, ID 900089285

MTH 213, Quiz 7

14.5/15 :)

Ayman Badawi

QUESTION 1. Out of 14 available persons (8 males and 6 females) a committee with 5 is formed. Assume that $f_1, f_2, ..., f_6$ are the names of the females and $m_1, m_2, ..., m_8$ are the names of the males.

(i) If f_3 and m_5 must be in the committee, in how many different ways can we form a such committee?

$\binom{1}{1}\binom{1}{1}\binom{12}{3}$ $1C1 \times 1C1 \times 12C3 = 220$ 2/2

(ii) If f_3, f_5, and exactly 2 males must be in the committee, in how many different ways can we form a such committee?

$\binom{1}{1}\binom{1}{1}\binom{8}{2}\binom{4}{1}$ $1C1 \times 1C1 \times 8C2 \times 4C1 = 112$ 2/2

(iii) If m_4 or m_7, but not both, must be in the committee, in how many different ways can we form a such committee?

$(m_4 \& -)$ or $(m_7 \& -)$

$\binom{1}{1}\binom{12}{4} + \binom{1}{1}\binom{12}{4}$ $(1C1 \times 12C4) + (1C1 \times 12C4) = 990$ 2/2

(iv) If exactly 3 females must be on the committee, in how many different ways can we form a such

$\binom{6}{3}\binom{11}{2}$ $6C3 \times 8C2 = 1100$ 1.5/2

QUESTION 2. The digits 1, 2, 3, ..., and 8 are used to construct 4-digits car plates.

(i) If two adjacent digits must be different, how many EVEN car plates can be constructed?

7 7 7 4

$7 \times 7 \times 7 \times 4 = \cancel{4088}$ 1372 2/2

(ii) If a digit cannot be repeated in a plate number, the first digit, and third digit must be odd digits, how many car plates can be constructed?

1st 3rd

4 6 3 5 no repeats

$4 \times 6 \times 3 \times 5 = 360$ 2/2

(iii) If a digit cannot be repeated in a plate number, exactly one of the digits must be an even number, how many ODD car plates can be constructed?

no repeat;

3 2 4 4 or 3 4 2 4 or 4 3 2 4

$(3 \times 2 \times 4 \times 4) + (3 \times 4 \times 2 \times 4) + (4 \times 3 \times 2 \times 4) = 28$: 2/2

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